

Weak binding limit and non zero angular momentum states in Light-Front Dynamics

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We show some results concerning the weak binding limit for $J = 0$ states – which turn out to strongly differ from the non relativistic case – together with the construction of non zero angular momentum states. The calculation of such states in the Light-Front Dynamics (LFD) framework has some peculiarities which are absent in other approaches. They are related to the fact that the rotation generators contain interaction. We present here the construction of non zero angular momentum states in LFD and show how it leads to a restoration of rotational invariance. For this purpose, the use of Light-Front Dynamics in its explicitly covariant formulation [1] is of crucial importance since the dependence of the wave function on the light-front plane is explicitly parametrized.

In the explicitly covariant version of LFD, the state vector is defined on a plane whose equation is written in the form: $\omega \cdot x = \sigma$, where ω is a light-like four-vector. The standard approach of LFD [2] is recovered for $\omega = (1, 0, 0, -1)$. The dynamical equations for the state vector describing a physical system are obtained by imposing that it obeys the Poincaré group transformation laws and that it belongs to an irreducible representation. In addition, we are interested in systems with definite four-momentum p^μ and angular momentum projection λ , that is Ψ satisfies:

$$\hat{P}^2 \Psi = M^2 \Psi, \quad \hat{P}^\mu \Psi = p^\mu \Psi, \quad (1)$$

$$\hat{S}^2 \Psi = -M^2 J(J+1) \Psi, \quad \hat{S}_3 \Psi = M \lambda \Psi. \quad (2)$$

\hat{P}^μ is the four-momentum operator and \hat{S}^μ the Pauli-Lubansky one, given by

$$\hat{P}^\mu = \int T^{\mu\nu}(x) \delta(\omega \cdot x - \sigma) \omega_\nu d^4x, \quad \hat{S}_\mu = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \hat{P}^\nu \hat{J}^{\rho\sigma}$$

where $T^{\mu\nu}$ is the energy momentum tensor and M the total mass of the system.

We consider a model consisting of two interacting scalar fields φ and χ with masses m and μ respectively [3]. The interaction Hamiltonian is $\mathcal{H} = -g\varphi^2\chi$. The wave functions are the Fock components of the state vector and the kernels are restricted to the usual ladder approximation. We are looking for solutions written on a form which directly satisfies the four-momentum eigenvalue equation in (1). The mass eigenvalue equation in (1) results into a three-dimensional integral equation for the two-body Fock component $\psi(\vec{k}, \hat{n})$:

$$[4(\vec{k}^2 + m^2) - M^2]\psi(\vec{k}, \hat{n}) = -\frac{m^2}{2\pi^3} \int \frac{d^3k'}{\varepsilon_{k'}} V(\vec{k}, \vec{k}', \hat{n}, M) \psi(\vec{k}', \hat{n})$$

\vec{k} is the momentum of one particle in the system of reference where $\vec{k}_1 + \vec{k}_2 = \vec{0}$, \hat{n} is the unit vector in the direction of $\vec{\omega}$ and $\varepsilon_k = \sqrt{\vec{k}^2 + m^2}$. For $J = 0$ state, ψ is a scalar function depending on the scalars k and $\hat{k} \cdot \hat{n}$. The kernel in the r.h.-side can be integrated analytically over the azimuthal angle and the equation is reduced to a two-dimensional one. The solutions for this model in

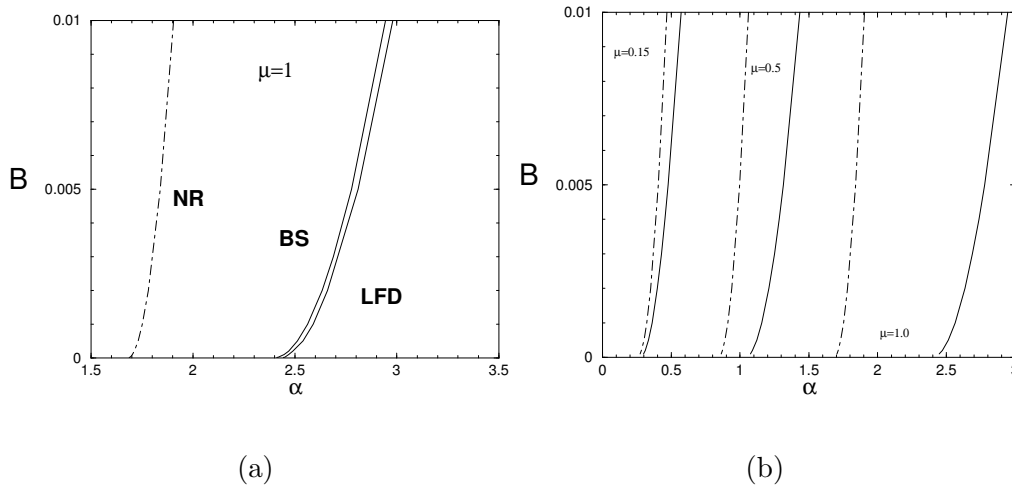


Fig. 1. Zero binding energy limit of LFD and BS equations (solid line) compared with the corresponding non relativistic solution (dot-dashed), for $\mu = 1$ (a) and for different values of μ (b).

LFD have been obtained in [3] and turn to be very close to those provided by Bethe-Salpeter equation in a wide range of coupling constant, well beyond the perturbative region. We found in particular (see figure 1), that for $\mu \neq 0$ and in the zero binding limit, the results for both equations – indistinguished in Figure (b) – dramatically depart from the non-relativistic theory, that is

Schrödinger equation with a Yukawa potential. This difference vanishes for $\mu = 0$ and increases with μ . We conclude from that to the irrelevance of a non-relativistic approach to describe systems interacting via massive fields, what is the case of all the strong interaction physics when not described by gluons.

Equations (2) determine the total angular momentum J and its projection λ . They contain interaction and this interaction dependence is not easy to deal with. To circumvent this difficulty, we make use of the so-called angular condition, derived from the transformation properties of the state vector under rotations of the light-front plane. In the same way the time dependence is given by Schrödinger equation, the dependence on the the light-front direction ω is given by [4]

$$\hat{L}_{\mu\nu}(\omega)\Psi_\omega(p) = \hat{J}_{\mu\nu}^{int}\Psi_\omega(p) \quad (3)$$

where $\hat{J}_{\mu\nu}^{int}$ is the interacting part of the angular momentum tensor $\hat{J}_{\mu\nu}$ and $\hat{L}_{\mu\nu}$ reads

$$\hat{L}_{\mu\nu}(\omega) = i \left(\omega_\mu \frac{\partial}{\partial \omega^\nu} - \omega_\nu \frac{\partial}{\partial \omega^\mu} \right)$$

Assuming that the state vector satisfies (3), we can replace in the Pauli-Lubansky operator $\hat{J}_{\mu\nu}^{int}$ by $\hat{L}_{\mu\nu}$. Namely, instead of the dynamical angular momentum tensor $\hat{J}^{\mu\nu}$ we introduce the operator $\hat{M}_{\mu\nu} = \hat{J}_{\mu\nu}^0 + \hat{L}_{\mu\nu}$. The set \hat{P}_μ and $\hat{M}_{\mu\nu}$ form an algebra of the Poincaré group in the same way as \hat{P}_μ and $\hat{J}_{\mu\nu}$ do. We thus construct a new Pauli-Lubansky vector: $\hat{W}_\mu = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\hat{P}^\nu\hat{M}^{\rho\sigma}$ and, provided Ψ verifies the angular condition, equations (2) can be replaced by

$$\hat{W}^2\Psi = -M^2J(J+1)\Psi, \quad \hat{W}^3\Psi = M\lambda\Psi \quad (4)$$

We deal now with an extended Fock space, where ω is not a fixed parameter but a four-vector variable. Since Ψ is an eigenvector of P^μ , equations (4) determining the angular momentum are *purely kinematical*. Therefore, their solutions can be found using the standard methods of the theory of angular momentum.

Now, to construct $J \neq 0$ states, we start solving the mass and four-momentum equations (1) together with equations (4). This procedure gives $2J+1$ solutions which, in a full Fock space, are degenerated in mass [1]. Indeed, it is readily verified that the operator $\hat{A}^2 = (\omega \cdot \hat{W})^2$ commutes with P^μ , W^2 , W_3 and with parity. Therefore the state vector satisfying (1) and (4) is characterized not only by its mass, momentum and angular momentum, but also by the

eigenvalues a^2 of \hat{A}^2 : $\hat{A}^2\Psi_a = a^2\Psi_a$. The operator \hat{A}^2 has $J + 1$ eigenvalues and $2J + 1$ eigenfunctions which are split in two families of opposite parities.

\hat{A}^2 commutes also with \hat{S}^2 but not with the operator $\hat{L}_{\mu\nu} - \hat{J}_{\mu\nu}^{int}$, that is the states Ψ_a do not satisfy the angular condition (3). The physical solution, i.e. the solution satisfying the angular condition, is constructed from a superposition of the degenerated states Ψ_a :

$$\Psi_{J\lambda} = \sum c_a \Psi_a \quad (5)$$

with coefficients c_a chosen such that $\Psi_{J\lambda}$ satisfies (3).

Let us summarize the way we construct states with definite angular momentum. Instead of dynamical equations (1) and (2), we solve equations (1) together with the kinematical ones (4). To have the full equivalence between both systems of equations, the solutions must be constructed as a superposition satisfying the angular condition (3).

We propose a simple method to find the right linear combination without explicitly solving (3). It is based on the fact that, when $k \rightarrow 0$, the interaction part $J_{\mu\nu}^{int}$ is irrelevant and the angular condition reads: $\hat{L}_{\mu\nu}\Psi = 0$. Thus in this limit, Ψ does not depend on the light-front plane direction and hence should not depend on \hat{n} anymore. This unambiguously determines the coefficients of the superposition.

Our method is hereafter explicated for the case $J = 1^-$ but the results will be given for $J = 2$ states as well. We are looking for solution of (1) and (4) with $J = 1^-$, which are also eigenvectors of \hat{A}^2 . The general form of these solutions reads $\psi_a(\hat{k}, \hat{n}) = \vec{\chi}_a(\hat{k}, \hat{n}) g_a(k, z)$ where $z = \hat{n} \cdot \hat{k}$ and $\vec{\chi}_a(\hat{k}, \hat{n})$ are given by

$$\vec{\chi}_0(\hat{k}, \hat{n}) = 3z\hat{n} \quad \vec{\chi}_1(\hat{k}, \hat{n}) = \frac{3\sqrt{2}}{2}(\hat{k} - z\hat{n})$$

The scalar functions $g_0(k, z)$ and $g_1(k, z)$ satisfy:

$$[4(\vec{k}^2 + m^2) - M_0^2]zg_0(k, z) = -\frac{m^2}{2\pi^3} \int \frac{d^3k'}{\varepsilon_{k'}} V(\vec{k}', \vec{k}, \hat{n}, M_0) z' g_0(k', z') \quad (6)$$

$$\begin{aligned} [4(\vec{k}^2 + m^2) - M_1^2](1 - z^2) g_1(k, z) \\ = -\frac{m^2}{2\pi^3} \int \frac{d^3k'}{\varepsilon_{k'}} V(\vec{k}', \vec{k}, \hat{n}, M_1) (\hat{k} \cdot \hat{k}' - zz') g_1(k', z') \end{aligned} \quad (7)$$

As a consequence of the Fock space truncation, the mass degeneracy of states with different a is violated. These states are split: different a 's correspond to different masses. Therefore equations (6) and (7) give two solutions with different masses M_0 and M_1 . It was found in [5] that the splitting reduces

when the two-boson exchanges, incorporating effectively higher Fock states, are added to the kernel. According to (5), the physical solution is

$$\vec{\psi}(\vec{k}, \hat{n}) = c_0 \vec{\psi}_0(\vec{k}, \hat{n}) + c_1 \vec{\psi}_1(\vec{k}, \hat{n}) \quad (8)$$

with c_0 and c_1 determined by the condition that in the vicinity of $k = 0$ $\vec{\psi}(\vec{k}, \hat{n}) \propto k$ is independent of \hat{n} , which leads to

$$c_0 = h_1 / \sqrt{2 + h_1^2} \quad c_1 = \sqrt{2} / \sqrt{2 + h_1^2}$$

with $h_1 = \lim_{k \rightarrow 0} \frac{g_1(k, z)}{g_0(k, z)}$. The coefficients c_0 and c_1 , which in principle depend on α , were found to be very close to $\sqrt{\frac{1}{3}}$ and $\sqrt{\frac{2}{3}}$ (with the accuracy $\approx 1\%$). We emphasize that we take the superposition (8) inspite of the fact that $\vec{\psi}_{0,1}$ correspond to different masses and the mass of the physical solution is

$$M^2 = c_0^2 M_0^2 + c_1^2 M_1^2$$

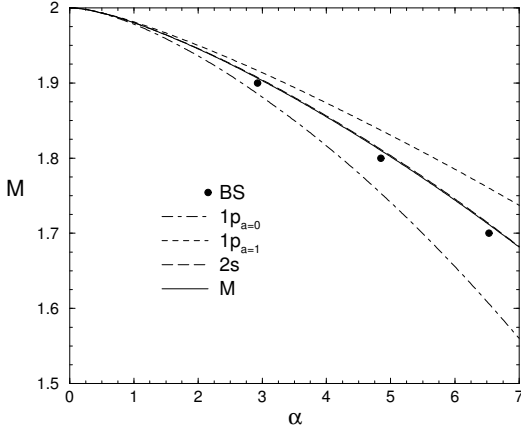


Fig. 2. $2s$ and $1p$ degeneracy ($\mu=0$)

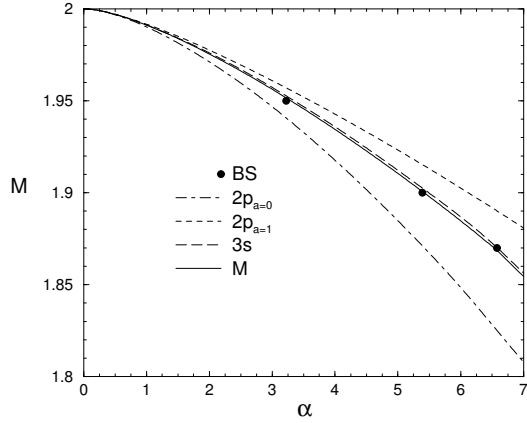


Fig. 3. $3s$ and $2p$ degeneracy ($\mu=0$)

In Figure 2 the masses of $2s$ state are shown, together with M_0 , M_1 and the physical mass of the $1p$ state M , as functions of the coupling constant $\alpha = g^2/16\pi m^2$. It is seen that the degeneracy which exists between $2s$ and $1p$ states in the case $\mu = 0$ is restored up to a high accuracy. This is also the case for $3s$ and $2p$ states in Figure 3. Furthermore the physical masses M are shown to be close to those provided by the resolution of Bethe-Salpeter equation, whereas the two approaches do not have the same diagrammatical content.

We display in Figure 4 the equivalent results for $J = 2$, i.e. the $3s$ - $1d$ degeneracy. Let us however mention that the case $J > 1$ has some specificities which require some care, and will be presented elsewhere.

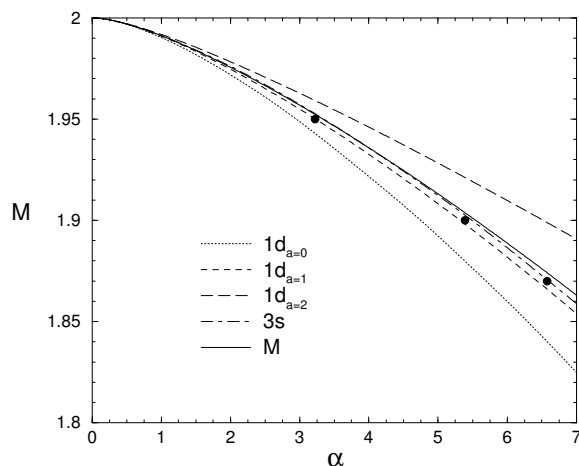


Fig. 4. Restoration of $3s$ and $1d$ degeneracy for $\mu = 0$

As a summary, by studying the zero binding limit, we come to the conclusion that composite systems strongly interacting via a massive exchange cannot be properly described by a non relativistic dynamics [3]. Furthermore, we have developed a method to construct non zero angular momentum states in the explicitly covariant LFD framework. The results presented for $J = 1$ and $J = 2$ show that the rotational degeneracy in the case of a massless exchange is restored up to a high level of accuracy, and that binding energies are close to Bethe-Salpeter ones. The construction of $J \neq 0$ states proposed has been applied to a scalar model: the next challenge is the description of fermionic systems.

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